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# Where is the Information Stored in Black Holes?

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## Abstract

It is shown that many modes of the gravitational field exist only inside the horizon of an extreme black hole in string theory. At least in certain cases, the number of such modes is sufficient to account for the Bekenstein-Hawking entropy. These modes are associated with sources which carry Ramond-Ramond charge, and so may be viewed as the strong coupling limit of D-branes. Although these sources naturally live at the singularity, they are well defined and generate modes which extend out to the horizon. This suggests that the information in an extreme black hole is not localized near the singularity or the horizon, but extends between them.

## I. INTRODUCTION

A key ingredient in understanding the black hole information puzzle [1] is the question of where the states accounting for the black hole entropy are located. If the Bekenstein-Hawking entropy [2,3] is associated with the matter that forms the black hole, then it would appear that these states are localized near the singularity. On the other hand, since the entropy is proportional to the horizon area, it has been suggested that these states are associated with horizon fluctuations [4]. The resolution is important for deciding whether information is lost in black hole evaporation, since storing the information near the singularity would make it difficult to be recovered without violating causality or locality. This, of course, would not be such a problem if information were stored near the horizon.

Recently, the states associated with extremal and near extremal black holes have been identified in weakly coupled string theory [5–8]. The number of such states exactly reproduces the black hole entropy (for large black holes). While there are plausible arguments for extrapolating the *number* of states from weak coupling to strong coupling, there has so far been little discussion of what these weak coupling states look like at strong coupling, where a large black hole is present.

One can find arguments which support either possibility, that these states are localized near the singularity or near the horizon. For example, at weak coupling, one considers bound states of D-branes [9,10] which carry the same charges as the black hole. Although the size of such bound states is not known, it is expected to be no larger than the string length, which is set by the string tension. At strong coupling, the event horizon is much larger than the string length. Since the D-branes carry the charge, and the source of the charge in the black hole is the singularity, the D-branes must lie at the singularity. Thus the bound states of D-branes should be localized to within a string length of the singularity. However, the validity of the laws of black hole thermodynamics suggest that the Bekenstein-Hawking entropy should be associated with states that are accessible to (i.e. can interact with) external probes [11]. This seems to imply that the states are localized near the horizon.

We will argue that there is another possibility which combines the desired features of both alternatives. We will show that the gravitational field has a large number of modes which exist only inside the horizon of an extreme black hole. Since there is a timelike singularity inside the horizon, it is perhaps not surprising that additional modes exist. What *is* surprising is that the modes we consider do not propagate into another asymptotically flat region of spacetime, but are entirely contained within the horizon. The radial profiles of these interior modes are fixed and they extend from the singularity to the horizon. We will also show that there is a well defined sense in which these modes are generated by sources living at the black hole singularity. Since these sources carry Ramond-Ramond charge, it is natural to interpret them as the strong coupling limit of D-branes. Thus although the D-branes live at the singularity, they couple to modes which extend out to the horizon and carry the information about their state of excitation.

The interior modes we consider differ from waves outside the horizon [12–15] in several

respects. The exterior waves are all homogeneous in the compactified dimensions of the spacetime, while the interior waves are not. There are thus many more waves which exist inside the horizon. Although we cannot do a precise counting at this time, it is clear that the number of these modes is sufficient to account for the Bekenstein-Hawking entropy (at least when the black hole carries unit fivebrane charge). More importantly, it has recently been shown that, although the metric for the exterior waves is continuous on the horizon, there is a mild curvature singularity there[1] [16]. The interior modes we discuss will leave the metric at least $C^2$ on the horizon, so that no curvature singularity is generated.

The outline of this paper is as follows. We begin in section II by reviewing the exterior solutions of the extreme black holes we wish to consider, and describe the simplest (homogeneous) interior solutions. In section III, we present the equations governing a general class of interior modes and show that these modes can be associated with sources that live on the singularity. In the next section we discuss stationary solutions of these equations, and the smoothness of the horizon. In section V we add time dependence and discuss the connection with black hole entropy. In particular, the states associated with oscillating D-branes are described. Some concluding remarks and open questions are contained in section VI. The details of the matching conditions at the horizon are contained in appendix A. As a first step toward understanding how these interior modes might be excited, we study an oscillating test string as it falls into the black hole in appendix B.

## II. HOMOGENEOUS INTERIORS

We begin our investigation of the region behind the event horizon by considering the simplest homogeneous cases. Such solutions may be obtained, for example, by analytic continuation of an exterior solution through the horizon. This of course requires the use of coordinates in which the metric is in fact analytic at the horizon.

Let us therefore begin by reviewing the exterior solutions. The low energy action for the type IIB string theory contains the terms (in the Einstein frame)

$$S = \frac{1}{16\pi G}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12}e^{\phi}\ \mathcal{H}^2\right) \tag{2.1}$$

where $\phi$ is the dilaton and $\mathcal{H}$ is the Ramond-Ramond three form. We will consider solutions on $T^5 \times \mathbf{R}^5$, which reduce to black holes in $4+1$ dimensions. Following the conventions of our earlier papers [14,15], we label the 4+1 asymptotically flat 'external' directions by coordinates $(x^i, t)$, and divide the five torus into an $S^1$ (labeled by the coordinate $z$) and a $T^4$ (labeled by the coordinates $y^i$). We take the $S^1$ to have coordinate length $L$ and the $T^4$

[1] Despite this singularity, the area of the horizon is well defined and agrees with the counting of D-brane states [14,15]. The physical significance of this singularity is being investigated.

to have coordinate volume $\mathsf{V}$. It is often useful to think of the solution as corresponding to a black *string* in $5+1$ dimensions, where the string lies along the $z$ axis. In such a picture the $T^4$ would be considered an 'internal' four torus. These solutions carry electric and magnetic charge with respect to the three form $\mathcal{H}$. They also carry momentum in the $z$ direction. At weak coupling, these charges are reproduced by D-onebrane and D-fivebrane sources, with the fivebranes lying in the $(t, z, y^i)$ 5+1 space and the onebranes lying in the $(t, z)$ plane.

Introducing the coordinates $u = t - z$ and $v = t + z$, the exterior black hole solution takes the form [17,6]

$$ds^2 = H_1^{1/4} H_5^{3/4} \left[ \frac{du}{H_1 H_5} (-dv + K du) + \frac{dy_i dy^i}{H_5} + dx_i dx^i \right] \tag{2.2}$$

$$e^{-2\phi} = \frac{H_5}{H_1} \tag{2.3}$$

$$\mathcal{H}_{auv} = H_1^{-2} \partial_a H_1, \qquad \mathcal{H}_{ijk} = -\epsilon_{ijkl} \partial^l H_5 \tag{2.4}$$

where $\epsilon_{ijkl}$ is the flat space volume form on the $x$ space (the indices $i, j, k, l$ in (2.4) refer to the $x$ space) and $a$ runs over all $x^i, y^i$. The functions $H_1$, $H_5$, and $K$ are given by

$$H_1 = 1 + \frac{r_1^2}{r^2}, \qquad H_5 = 1 + \frac{r_5^2}{r^2}, \qquad K = \frac{p}{r^2} \tag{2.5}$$

where $r^2 = x_i x^i$, and the constants $r_1$, $r_5$, and $p$ determine the electric and magnetic charges of the three form, and the momentum respectively. In this coordinate system, the horizon lies at the coordinate singularity $r = 0$. Its area is given by $A = 2\pi^2 r_1 r_5 \mathsf{V} L \sqrt{p}$. The nonextremal black hole solutions are also known [18,19] but will not be considered here.

New coordinates may be introduced such that the metric is analytic at the horizon (see e.g. the appendix of [14]). It can then be continued into the interior. A further change of coordinates places the interior metric in the convenient form:

$$ds^2 = H_1^{1/4} H_5^{3/4} \left[ \frac{du}{H_1 H_5} \left( dv + \frac{p}{r^2} du \right) + \frac{dy_i dy^i}{H_5} + dx_i dx^i \right] \tag{2.6}$$

with *exactly* the same coordinate identifications (corresponding to the compact directions) as in (2.2). However, we now have

$$H_1 = \frac{r_1^2}{r^2} - 1, \qquad H_5 = \frac{r_5^2}{r^2} - 1. \tag{2.7}$$

Note that, aside from the form of the harmonic functions $H_1$ and $H_5$, the only difference between (2.6) and (2.2) is a single negative sign in the $dudv$ term. As a check on these formulas, note that for the case $r_1 = r_5 \equiv r_0$, both metrics (2.6) and (2.2) reduce to the more familiar form of the solution

$$ds^2 = -\left(1 - \frac{r_0^2}{\hat{r}^2}\right) dudv + \frac{p}{\hat{r}^2} du^2 + \left(1 - \frac{r_0^2}{\hat{r}^2}\right)^{-2} d\hat{r}^2 + \hat{r}^2 d\Omega_3^2 + dy_i dy^i \tag{2.8}$$

where $r^2 = \hat{r}^2 - r_0^2$ in the exterior and $r^2 = r_0^2 - \hat{r}^2$ in the interior.

In the interior metric (2.6) the horizon again lies at $r = 0$. Thus, this coordinate system is 'inside-out' in the sense that moving to larger values of $r$ corresponds to moving deeper into the interior. Note that the singularity lies at $r = r_1$ or $r = r_5$ (whichever is smaller) where $H_1$ or $H_5$ vanishes. This is a timelike curvature singularity, much like the singularity of usual extreme Reissner-Nordström black holes. As a result of the change of sign in the $dudv$ term, the coefficient of $dz^2$ is proportional to $(\frac{p}{r^2} - 1)$ which becomes negative for $r > \sqrt{p}$. Thus, if $\sqrt{p} < r_1, r_5$ the physical region $0 < r < r_1, r_5$ contains closed timelike curves.

## III. MORE GENERAL INTERIOR SOLUTIONS AND SOURCES

Although the interior solution described above is the unique analytic extension of the exterior solution, it turns out that there are many other interior solutions which leave the horizon nonsingular. We now discuss these more general solutions and show that they can be viewed as arising from sources at the singularity. Since these sources carry RR charge, they are naturally interpreted as the strong coupling limit of D-branes.

We will consider solutions of the form

$$ds^2 = H_1^{1/4} H_5^{3/4} \left[ \frac{du}{H_1 H_5} (\epsilon dv + K du + 2A_i dy^i) + \frac{dy_i dy^i}{H_5} + dx_i dx^i \right] \tag{3.1}$$

$$e^{-2\phi} = \frac{H_5}{H_1} \tag{3.2}$$

$$\mathcal{H}_{auv} = H_1^{-2} \partial_a H_1, \qquad \mathcal{H}_{aub} = 2\partial_{[a} A_{b]}, \qquad \mathcal{H}_{ijk} = -\epsilon_{ijkl} \partial^l H_5 \tag{3.3}$$

where $H_1 = H_1(u, x, y)$, $H_5 = H_5(x)$, $K = K(u, x, y)$, $A_{y^i} = A_{y^i}(u, x, y)$, and the indices $a, b$ run over $x^i, y^j$ though $A_{x^i} = 0$. This is a generalization of solutions that have been considered previously [20–24]. This ansatz preserves the null translational symmetry $\partial/\partial v$ of the original solution. Here $\epsilon$ is a sign which is clearly arbitrary (as it may be changed by sending $v \to -v$), but which must be reversed (as in section II) when matching interior and exterior solutions. The motivation for this form of the solution comes from the description of the microstates of this black hole in the limit of weak string coupling. For a single fivebrane, the entropy comes from the oscillations of onebranes inside the fivebrane. Since $H_5$ is associated with the fivebrane (this will be made more precise below) which does not oscillate, we have assumed that it is only a function of $x$. The vector $A_i$ describes momentum flow in the $i^{th}$ direction. Since the onebranes only oscillate inside the fivebrane, we have assumed that $A_i$ has components only in the internal ($y$) directions.

It may be verified that (3.1)-(3.3) is an extremum of the action (2.1) when the following five conditions are satisfied:

$$
\begin{aligned}
\partial_x^2 H_5 &= 0 \\
\partial_x^2 H_1 + H_5 \partial_y^2 H_1 &= 0 \\
\partial_x^2 K + H_5 \partial_y^2 K + 2H_5 \partial_u [\partial_u H_1 - \partial_i A^i] &= 0 \\
\partial_{x^j} [\partial_u H_1 - \partial_i A^i] &= 0 \\
\partial_x^2 A_i + H_5 \partial_y^2 A_i + H_5 \partial_i [\partial_u H_1 - \partial_j A^j] &= 0
\end{aligned}
\tag{3.4}
$$

where the indices $i, j$ run over the four $y$ coordinates, $\partial_x^2$, $\partial_y^2$ are the *flat-space* Laplacians associated with the $x$ and $y$ coordinates, and the indices $i$ are raised and lowered using the flat space Euclidean metric. In particular, when $\partial_u H_1 = \partial_i A^i$ and the solutions are independent of $y$, we find that $H_1$, $H_5$, $K$, and $A_i$ are just flat-space harmonic functions of $x$. Given that the null symmetry $\partial/\partial v$ is preserved, it is likely that these solutions are supersymmetric.

Since we can solve for $H_5$ first, the above equations are all essentially linear. As a result, we may think of these fields as produced by a set of localized sources which lie at the singularity. Although the singularity is pointlike in the physical spacetime metric, it corresponds to the vanishing of $H_1$ or $H_5$, which often occurs on a surface of finite coordinate size in the four dimensional Euclidean space parameterized by $x$. In electrostatics, if one wants to solve Poisson's equation with a shell of charge, one usually demands regularity at the origin, so that the solution is trivial inside and nontrivial outside the shell. Here, the requirement of an event horizon requires that the solution be nontrivial inside the shell, so we can take it to be trivial outside. For example, consider the homogeneous interior solution (2.6), (2.7) with $r_1 < r_5$, so the singularity is at $r = r_1$. Away from the singularity, $r < r_1$, this solution is identical to the solution $A_i = 0$,

$$
H_1 = \left(\frac{r_1^2}{r^2} - 1\right)\theta(r_1 - r), \quad H_5 = \left(\frac{r_5^2}{r^2} - 1\right)\theta(r_1 - r) + \left(\frac{r_5^2}{r_1^2} - 1\right)\theta(r - r_1)
$$

$$
K = \left(\frac{p}{r^2}\right)\theta(r_1 - r) + \left(\frac{p}{r_1^2}\right)\theta(r - r_1) \tag{3.5}
$$

where $\theta(r)$ is the usual step function. The only difference is that the solution (3.5) has sources at $r = r_1$ with just the right strength to account for the total charges and momentum of the black string. Since the source of the charges for the black string should lie at the singularity, (3.5) is a more accurate description of the physics. As the sources carry RR charge, they can be viewed as the strong coupling limit of D-branes.

One might ask what would happen if one tried to place the sources away from the singularity at some $r = r_0 < r_1$. This turns out to be unphysical since the sources would have negative local energy density. This can be seen as follows. If the sources are at $r = r_0 < r_1$, then the spacetime is nonsingular and $H_1$, $H_5$, and $K$ are all positive *constants* for $r > r_0$. Hence, this region of spacetime is completely flat. One thus obtains a nonsingular solution

with zero total energy and an event horizon. The positive energy theorem for black holes [25] states that any such spacetime must contain matter with negative local energy density. The reason for the negative energy density can be understood physically by considering the four dimensional example of a shell of $q = m$ dust. The spacetime is the extreme Reissner-Nordström solution with charge $Q$ outside the shell and flat space inside. When the shell is large, the energy in the electromagnetic field is small and most of the energy comes from the shell. As we decrease the radius of the shell, the total charge (and hence total mass) remains constant, but the energy in the electromagnetic field increases. Thus the energy density of the shell must decrease. For small enough shells (inside the horizon of the extreme black hole) the energy density must become negative. This may be studied in detail by using the spherically symmetric mass function appropriate to this spacetime.

This result has implications for which D-branes can be placed in static equilibrium around an extreme (positively charged) black hole. Outside the horizon, there exist static, BPS solutions with D-branes which have positive charge and energy density. There are also static solutions with sources that have negative charge and negative energy density, but these are not usually considered because they are unphysical. Inside the horizon, the situation is reversed. It is the negatively charged D-branes which remain static (and have positive energy density). A positively charged source could remain static inside the horizon only if it had negative energy density.

The idea that a given set of fields ($H_1$, $H_5$, $K$, and $A_i$) may be thought of as being produced by sources at the singularity in fact holds much more generally. The trick is to choose the proper boundary conditions as, in general, the solutions to (3.4) will not all be constant at the singularity, so they cannot be extended as constants beyond the singularity. Let us assume $\partial_u H_1 = \partial_i A^i$. We will see later that these are the solutions of greatest interest. Then $H_1$, $K$, and $A_i$ are all determined by the same differential operator

$$\tilde{\nabla}^2 = \partial_x^2 + H_5 \partial_y^2. \tag{3.6}$$

The field between the horizon and the singularity can be associated with a unique set of sources at the singularity (and a unique set of fields in the unphysical region) provided that the equations

$$\begin{aligned} \tilde{\nabla}^2 H_1 &= 0 \\ \tilde{\nabla}^2 K &= 0 \\ \tilde{\nabla}^2 A_i &= 0 \end{aligned} \tag{3.7}$$

are imposed everywhere except on the surface where the charge will lie (the surface where $H_1$ or $H_5$ first vanishes) and provided that the fields are required to approach constant values at large $r$ and to carry no net (monopole) charge; i.e. $H_1 \sim const + \mathcal{O}(r^{-3})$.

Two comments about this association between fields and sources are now in order. The first is that, in general, the above procedure may not preserve the condition $\partial_u H_1 = \partial_i A^i$ in the unphysical region beyond the singularity. As a result, they are not solutions to the full equations of motion in this region. However, there is no reason why the full equations

of motion must be satisfied in the region beyond the singularity. If one wanted to keep the equations satisfied, there are two possibilities. Since $A_i$ describes the momentum of the sources, roughly speaking the problem is that some interior solutions describe sources which remain on some surface $S$ but have momentum transverse to this surface. This may be remedied by allowing the sources of the $A_i$ fields to extend off of the singular surface, deeper into the unphysical region. A simple counting of degrees of freedom suggests that it can also be remedied by generalizing our solutions to include an $A$ field that also points in the $x$ directions (allowing oscillations in the $x$ directions) and by considering solutions with $\partial_u H_1 \neq \partial_i A^i$. However, for the purposes of the present work it will be sufficient to restrict attention to solutions for which the above procedure does in fact generate fields which satisfy $\partial_u H_1 = \partial_i A^i$, even in the unphysical region. This is because the most interesting solutions will in fact have this property.

The other comment concerns the boundary conditions imposed at large $r$. Since this condition refers to the region beyond the singularity, it is somewhat arbitrary. However, the fact that we must introduce a boundary condition by hand is not unexpected – it is simply due to the presence of the timelike singularity. We require the fields to approach constants as this is a familiar boundary condition for elliptic equations and will generate familiar relationships between the associated charge densities and fields. The zero net charge restriction says that the total charge found inside the horizon (i.e., in the $r > 0$ region) is equal to the total charge registered on the horizon. Finally, note that the interpretation of the fields as arising from sources at the singularity holds for a wide class of boundary conditions. It is only the exact form of the pairing between fields and sources that depends on the particular boundary condition chosen.

## IV. INHOMOGENEOUS INTERIORS

In this section we investigate interior solutions which are stationary, i.e. $u$ independent, but inhomogeneous in the $y$ directions. The effects of allowing $u$ dependence will be considered in the next section. We will set $\partial_i A^i = 0$, so the equations (3.4) decouple. We also assume that all fields remain spherically symmetric in the four dimensional $x$ space. This implies that $H_5$ keeps the same form that it had in the homogeneous case: $H_5 = \frac{r_5^2}{r^2} - 1$.

The functions $H_1$, $K$, and $A_i$ then satisfy (3.7). Let us examine the behavior of the solutions near the horizon of the black hole, $r = 0$. The coefficient $C_k(r)$ of the Fourier mode $e^{ik_j y^j}$ satisfies

$$r^{-3}\partial_r \left( r^3 \partial_r C_k \right) = k^2 \frac{r_5^2}{r^2} C_k \tag{4.1}$$

so that $C_k \sim r^\alpha$ where $\alpha = -1 \pm \sqrt{1 + k^2 r_5^2}$. The exact solutions can be expressed in terms of Bessel functions, although the detailed form of the solution will not be important here.

For $k^2 \neq 0$, the modes with $\alpha = -1 - \sqrt{1 + k^2 r_5^2} < -2$ cause the horizon to become singular. If such modes are present in the $H_1$ field, then since $e^\phi = \sqrt{H_1/H_5}$, such solutions

have divergent dilaton on the horizon. If such modes are present in $K$, then the norm of the Killing field $\partial/\partial u$ diverges on the horizon. If they are present in $A_i$ then, at the very least, the structure of the horizon is radically altered from the original homogeneous case. It seems likely that the horizon would again become singular. On the other hand, the modes with $\alpha = -1 + \sqrt{1 + k^2 r_5^2}$ vanish at the horizon. As a result, they do not alter the properties of the horizon, at least to leading order. In particular, the horizon itself remains homogeneous. A more careful study of such modes is performed in appendix A where it is found that a mode which behaves as $e^{iky}$ (and thus has $\alpha = -1 + \sqrt{1 + k^2 r_5^2}$) is at least $C^{\epsilon/2}$ for all $\epsilon < \alpha/2 - 1$ at the horizon. Thus, if any mode with $\alpha > 6$ is added to the interior solution, the horizon remains at least a $C^2$ surface even when the exterior spacetime is unchanged. In fact, for macroscopic black holes (with $r_5$ sufficiently large), *all* inhomogeneous Fourier modes become very smooth at the horizon. Thus, not only are such modes allowed but, for large $k^2 r_5^2$, they are not restricted by the exterior form of the black string. This differs from the case of the homogeneous modes with $k^2 = 0$ which must be matched across the horizon.

Can these inhomogeneous modes exist outside the horizon? Near $r = 0$, $H_5 \approx r_5^2/r^2$ both outside and inside the horizon. So exterior solutions again satisfy (4.1) and have the same behavior $r^\alpha$ near the horizon. However we must now consider the behavior of these modes far away from the horizon. While we could work with the exact expressions in terms of Bessel functions, it is sufficient to note that general arguments imply that any solution which is sufficiently regular at the origin must be appropriately singular at infinity[2]. Since the mode with $\alpha = 0$ behaves as a constant near infinity (in fact, it is just the constant solution), it follows that all modes with $\alpha > 0$ in fact diverge at infinity. Conversely, the modes with $\alpha < -2$ are well behaved near $r = \infty$, but of course diverge at the horizon.

Hence inhomogeneous modes are not allowed outside of the horizon, unless they are directly generated by sources in the exterior region. For example, if we add a single one brane at $(x_0, y_0)$ by solving

$$\tilde{\nabla}^2 H_1 = \delta(x - x_0)\delta(y - y_0) \tag{4.2}$$

with appropriate boundary conditions, then the field for $|x| < |x_0|$ would be composed of modes with $\alpha > 0$ while the field for $|x| > |x_0|$ would contain modes with $\alpha < 0$.

Inside the horizon, the fact that the inhomogeneous modes diverge at infinity is not a problem, since this occurs in the unphysical region beyond the singularity. Moreover, as discussed in the previous section, the behavior of these modes in the unphysical region is modified by sources at the singularity.

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[2] This follows from the fact that $\partial_x^2$ is a negative definite operator on $L^2(\mathbf{R}^4)$ while $k^2 H_5$ is positive.

## V. INTERIORS WITH WAVES

We now consider dynamical solutions to (3.4) where any of the various fields (except $H_5$) may depend on $u$ as well as the coordinates $x$ and $y$. Roughly speaking, each of the stationary modes described in the previous section may be given arbitrary $u$ dependence and so becomes a propagating mode. It is convenient to set

$$\partial_u H_1 = \partial_i A^i \tag{5.1}$$

so that (3.4) again simply reduces to the statement that $H_1$, $K$ and $A_i$ are annihilated by the operator $\tilde{\nabla}^2$ (3.6). The condition (5.1) is a natural one if we seek solutions whose sources can be interpreted as (now wiggling) D-branes. This is because one can view the source of $A_i$ as the momentum of the various branes while the source of $H_1$ is the energy density of the branes. As a result, (5.1) is like a continuity equation. In particular, the oscillating string sources of [23] are consistent with this condition.

To illustrate this, let us consider the solution corresponding to adding a single D-onebrane oscillating outside the horizon of the homogeneous black string background described in section II. To do so, we introduce a Green's function $\Lambda(x_1, y_1, x_2, y_2)$ for the elliptic operator $\tilde{\nabla}^2$; that is, $\Lambda$ satisfies

$$\tilde{\nabla}^2 \Lambda(x, y, x_0, y_0) = \delta(x - x_0)\delta(y - y_0) \tag{5.2}$$

and vanishes for large $x^2$. Then the change in the solution due to adding the onebrane is given by

$$\begin{aligned} \Delta H_1(x, y, u) &= m\Lambda(x, y, x_0, y_0(u)) \\ \Delta A^i(x, y, u) &= -m\dot{y}_0^i(u)\Lambda(x, y, x_0, y_0(u)) \\ \Delta K(x, y, u) &= m\dot{y}_0^2(u)\Lambda(x, y, x_0, y_0(u)). \end{aligned} \tag{5.3}$$

and describes a onebrane at $x = x_0$, $y = y_0(u)$. Such a solution carries the appropriate energy and momentum for an oscillating string of tension $m/\kappa^2$ [14] where $\kappa^2 = (2\pi)^5 g^2/\mathsf{V}$ and $g$ is the asymptotic string coupling. Note that (5.3) is an obvious generalization of the oscillating string solutions of [23]. The interpretation of this field as arising from the oscillations of a onebrane in fact forces the relation (5.1) between $A_i$ and $H_1$. This is just the statement that the momentum carried by a string is determined by its motion.

We can now describe a set of modes in the interior which seem to be a strong coupling analogue of the D-brane states which represent the black hole entropy at weak coupling. To begin, we recall that the integer normalized charges $Q_1$, $Q_5$, and $n$ are related to the parameters $r_1$, $r_5$, and $p$ by [19]

$$Q_1 = \frac{r_1^2\, \mathsf{V}}{(2\pi)^4 g}, \qquad Q_5 = \frac{r_5^2}{g}, \qquad n = \frac{pL^2\, \mathsf{V}}{(2\pi)^6 g^2} \tag{5.4}$$

We will consider the case $Q_5 = 1$, so that the weak coupling description of the black hole microstates consists of $Q_1$ onebranes oscillating inside a single fivebrane. To obtain the

strong coupling description of these states, we will choose sources for (3.4) which reproduce this behavior. This argument cannot, of course, be considered an independent derivation of the counting, but does serve to give a definite interpretation of the modes in the strong coupling regime.

To understand the fields generated by an oscillating onebrane, we cannot use (5.3) since this applies only to strings oscillating outside the horizon. It is possible to write down the analog of (5.3) for a onebrane oscillating inside the horizon. We must, of course, take care to impose the appropriate boundary condition stated in section III; in particular, in the asymptotic region beyond the singularity, the solution should approach a constant with zero total charge. (This insures that the charge seen at the horizon is given by the sources inside.) Since the homogeneous mode is fixed by matching to the exterior solution, we must also insure that the only change in this mode at $r = 0$ is the increase in charge and momentum due to the string. The result is that the change in the solution for a onebrane oscillating inside the horizon is

$$\begin{aligned} \Delta H_1(x,y,u) &= \frac{m}{r^2} - m\Lambda(x,y,x_0,y_0(u)) + m\Lambda(0,0,x_0,y_0(u)) \\ \Delta A^i(x,y,u) &= -m\dot{y}_0^i(u)\left[\frac{1}{r^2} - \Lambda(x,y,x_0,y_0(u)) + \Lambda(0,0,x_0,y_0(u))\right] \\ \Delta K(x,y,u) &= m\dot{y}_0^2(u)\left[\frac{1}{r^2} - \Lambda(x,y,x_0,y_0(u)) + \Lambda(0,0,x_0,y_0(u))\right]. \end{aligned} \tag{5.5}$$

Note that these fields satisfy $\partial_u H_1 = \partial_i A^i$ everywhere. As discussed earlier, since these sources have positive charge and are at fixed radius inside the horizon, they must have negative energy density. This need not concern us, since we will eventually place them at the singularity.

Since the equations are linear, there is no difficulty considering $Q_1$ different oscillating strings. To insure that the homogeneous modes are independent of $u$ (which is needed to keep the horizon regular [16]) we require that their oscillations combine in such a way that $\sum_A m_A\dot{y}_A^2(u)$ and $\sum_A m_A\dot{y}^i(u)$ (where $A$ runs over the various branes) are in fact independent of $u$.

To obtain the complete solution, we start with $H_5(r) = (\frac{r_5^2}{r^2} - 1)\theta(r_5 - r)$ which describes a fivebrane at $r = r_5$. We will assume that the singularity lies at $r = r_5$, i.e., $H_1$ will be nonzero for all $r < r_5$. This will be true whenever $r_1 > r_5$ (in the absence of oscillations), and the inhomogeneities are small. For most D-brane states, this is indeed the case[3]. We then assume that the source for $H_1$ at the singularity is composed of $Q_1$ separate pieces (onebranes) each carrying a unit quantum of charge. We assume these onebranes can oscillate

[3]In particular, if one counts only the weakly coupled D-brane states in which the onebranes all oscillate in nearly the same way, one still reproduces the Bekenstein-Hawking entropy. This equipartition theorem-like result follows, for example, from section III.B. of [14].

in the $y$ directions (since they are bound to the five brane at weak coupling) and hence they are described by four bosonic functions (the $y_0^i(u)$ above) which give the position of each onebrane. These sources generate nonzero fields $A_i$ and $K$ as well as $H_1$. In this way, the $4Q_1$ left-moving bosonic modes which account for the black hole entropy can be represented by modes living inside the black hole horizon. Although the sources live at the singularity, the modes extend out to the horizon. We have described these modes classically, but since they satisfy linear equations, it should be straightforward to quantize them and obtain a direct correspondence between their states and the quantum states of the D-branes.

One can ask whether these interior waves are restricted by the fact that there are no analogous waves in the exterior. This question is studied in detail in appendix A and the answer is no. As we have seen, these inhomogeneous modes all vanish near the horizon. The fact that the waves do not cross the horizon can be understood by considering the general metrics (3.1), with or without waves. Such solutions have a null Killing field $\partial/\partial v$ whose integral curves are null geodesics. The waves can be thought of as following these geodesics, which never cross the horizon. On the other hand, the homogeneous Fourier mode *is* restricted by the exterior solution. This follows simply from the fact that the area of the horizon is completely determined by this mode. When the homogeneous mode is independent of $u$, it is completely determined by continuity at the horizon.

It is interesting to ask whether the entropy can be counted directly from the low energy field theory without resorting to the D-brane analysis to fix the sources at the singularity. (For other approaches to this question, see [12,13,24,26].) Naively, there are an infinite number of modes since the wave number $k$ can be arbitrarily large. However, since we have been solving the low energy string equations, it is natural to count only modes with wavelength larger than the string scale. A heuristic counting of modes (for the case of a single fivebrane) which yields the right order of magnitude is the following. There are six fields ($H_1$, $K$, and $A_i$) which are roughly independent components of the solution. At the order of magnitude level, we can replace this set by a single field $\phi$ which satisfies the equation

$$\tilde{\nabla}^2\phi = 0. \tag{5.6}$$

Since the $u$ dependence of $\phi$ is unconstrained, the space of solutions reduces to a single left-moving $1+1$ scalar field for every allowed solution to (5.6) in the transverse $(x, y)$ space. Since (for macroscopic black strings) all solutions allow for quite smooth horizons, we shall include all solutions below our string scale cutoff. More precisely, we shall include all solutions for which the wave number $|k|$ is small enough that the wavelength of the mode on the internal torus at the horizon is above the string scale in the string metric: $g^S_{\mu\nu} = e^{\phi/2}g_{\mu\nu}$. The number of such modes is given by the volume $V_H$ of the internal four-torus at the horizon in the string metric in units of the string length, which is in turn determined (see 3.1 and 5.4) by the number of D-branes: $V_H = (2\pi)^4 Q_1/Q_5$. Since $Q_5 = 1$ for the case in question, we find on the order of $Q_1$ left-moving bosonic field modes. The exact D-brane counting gives $4Q_1$ such modes, which agrees at the order of magnitude level.

In the above counting of modes, we assumed that all of the modes were spherically symmetric. It is interesting to note, however, that there are aspherical solutions to the above equations as well. In fact, such solutions behave much like the ones that we have already discussed. For sufficiently high angular momentum, or when the mode is also inhomogeneous in the internal ($y$) directions, such modes are again arbitrarily smooth on the horizon. It is tempting to try to associate these modes with the $Q_5$ factor in the entropy when $Q_5 > 1$, but it is not yet clear how this will come about.

## VI. DISCUSSION

We have studied the region inside the horizon of an extreme black hole in string theory. For the same exterior geometry, we have found a large number of interior solutions which leave the horizon nonsingular. These solutions contain modes which propagate entirely inside the horizon, and can be viewed as generated by sources living on the singularity. We have seen that one can choose these sources to behave like weakly coupled D-branes, and thus obtain a strong coupling description of these states. In this way, one avoids the usual conflict over whether the information is localized near the singularity or the horizon, since these modes extend from one to the other.

There are several questions which remain open. One of them is whether it is possible to improve the counting of these modes and precisely reproduce the black hole entropy directly from the low energy field theory. To do this, one must understand the role of nonspherically symmetric modes. In our rough counting in the previous section, we included only spherically symmetric states despite the fact that (even for $Q_5 = 1$) the sphere at the horizon is large at large coupling, so that nonspherical modes can be both smooth and have wavelengths much larger than the string scale at the horizon. Another question involves how to extend this counting to the case with $Q_5 > 1$. We have focused on the solutions with RR charges, but it is clear from S-duality that there are analogous solutions (with the same number of modes propagating inside the horizon) carrying NS charges[4]. If the counting of these modes can be shown to reproduce the Bekenstein-Hawking entropy, then this would provide an explanation of the entropy of black holes with NS charges as well.

Although we have discussed only extremal black holes, it is likely that a near extremal black hole will have similar modes which may not persist indefinitely, but will be very long lived. An object which falls into an extreme black hole is likely to excite these modes. What effect do these interior modes have on Hawking radiation? Can the information about what falls in now be recovered in the Hawking evaporation?

To begin to address these questions, one can consider a single oscillating (positively charged) D-string falling into a black string. There are static solutions with the D-string

[4]We thank A. Tseytlin for this observation.

oscillating at any radius outside the horizon (given by (5.3)), but not inside. As we discussed in section II, the static positively charged solutions require a negative energy density inside the horizon. If the energy density and charge are both positive, the string will experience a repulsive force. In Appendix B we study the motion of an oscillating test string and find that the string falls smoothly through the horizon, reaches a minimum radius and expands out into another asymptotically flat region of spacetime. The state of oscillation remains completely unchanged. To show that the oscillating string excites the interior modes requires going beyond the test string approximation.

Something unusual may happen at the event horizon when a nontest string approaches. Recall that all nonsingular inhomogeneous modes vanish at the horizon. The horizon itself always remains homogeneous. It is not yet clear whether this is just a property of the modes we have considered (which all preserve a null translational symmetry, and are likely to be supersymmetric) or whether this property holds more generally. If it does, any perturbation outside the horizon which is inhomogeneous in the compact directions, must become homogeneous when it crosses the horizon. It is then likely that the perturbation would remain homogeneous inside, and could not excite the interior modes. However, in this case, it would appear that the horizon must become singular when a onebrane passes through – not just at the particular point $y = y_0$ occupied by the onebrane on the horizon, but over the entire four-torus. Perhaps a more plausible alternative is that inhomogeneities are not always smoothed out when an object crosses the horizon, due to transient modes which do not preserve a null translational symmetry. In this case, the interior modes that we have discussed can be excited.

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## APPENDIX A: THE SMOOTHNESS OF THE HORIZON

In this appendix we address the question of to what extent our various modes in the interior of the black string attach smoothly to the exterior. Recall [14] that the smoothness of the horizon is difficult to analyze when the homogeneous modes (that is, the $y$ translationally invariant and spherically symmetric modes such as $K = p/r^2$) are $u$-dependent. In fact, it now seems [16] that the horizon is actually singular when such modes are nontrivial functions of $u$. However, since we are primarily interested in the behavior of the higher modes, this will not cause a problem; we simply take the homogeneous modes to be independent of $u$

while allowing arbitrary $u$ dependence for the higher modes. Note that dropping only the lowest modes does not affect the counting of D-brane states.

For simplicity, we shall study the case with $r_1 = r_5 = r_0$, although the more general case may be addressed by the same techniques and yields corresponding results. We are interested in the effect of adding a term $\Delta$ to the field $H_1 = \frac{{r_0}^2}{r^2} - 1$ such that $\Delta \sim C_\Delta(u) r^\delta$ for $\delta > 0$ near $r = 0$. We will also add a similar term $k \sim C_k(u) r^\kappa$ to $K = \frac{p}{r^2}$ and a term $a_i \sim C_{a_i}(u) r^{\alpha_i}$ to $A_i = \frac{r_0^2 \dot{f}_i}{r^2}$ ($\kappa, \alpha_i > 0$). Here, $\dot{f}_i = df_i/du$ but since we keep the homogeneous modes independent of $u$, $\ddot{f}_i = 0$ and $f_i(u) = u\dot{f}_i$. We use this notation to coincide with that of [14]. Note that due to the compactness of the $S^1$ ($z$) direction, the coefficients $C_\Delta(u)$, $C_k(u)$, and $C_{a_i}(u)$ are periodic in $u$. As a result, they are bounded but approach no well-defined limit at the horizon ($u \to -\infty$).

A final simplification will be to conformally transform the metric by multiplying by $e^{-\phi/2}$. If both $e^\phi$ and the new metric are $C^n$ (and $\phi$ is finite), then it follows that the original metric is $C^n$ as well. The particular metric to be analyzed is then[5]

$$ds^2 = H_1^{-1} du \left( dv + K du + 2A_i dy^i \right) + dy^2 + H_5 dx^2; \tag{A1}$$

that is,

$$ds^2 = \left( \frac{r_0^2}{r^2} - 1 + \Delta \right)^{-1} du \left( dv + \left( \frac{p}{r^2} + k \right) du + \left( 2r_0^2 \frac{\dot{f}_i}{r^2} + 2a_i \right) dy^i \right) + dy^2 + H_5 dx^2. \tag{A2}$$

Note that the $A_i = \frac{r_0^2 \dot{f}}{r^2}$ term corresponds to the 'internal' waves of [14], although our $y$ coordinates are the $y'$ coordinates of [14]. Note also that our $p$ would be $p - r_0^2 \dot{f}^2$ in the notation of [14,15].

In analogy with the procedure followed in [14], we now introduce new coordinates $R = r_0 \sqrt{\frac{r_0^2 - r^2}{r^2}}$, $\hat{y}^i = y^i + f^i$ and $\hat{v} = v + 2\dot{f}_i \hat{y}^i - \int du \dot{f}^2$ so that the metric becomes

$$\begin{aligned} ds^2 = r_0^2 \Big\{ & \left( R^2 + r_0^2 \Delta \right)^{-1} du \Big( d\hat{v} + (2a_i - 2\Delta \dot{f}_i) d\hat{y} \\ & + \left( (p - r_0^2 \dot{f}^2) \frac{R^2 + r_0^2}{r_0^4} + k + \Delta \dot{f}^2 - 2a_i \dot{f}^i \right) du \Big) + r_0^{-2} d\hat{y}^2 + R^{-2} Z^{-3} dR^2 + Z^{-1} d^2\Omega_3 \Big\}. \end{aligned} \tag{A3}$$

A further change of coordinates to $U = \frac{1}{2\sigma} e^{2\sigma u}$, $V = \hat{v} + 4r_0^2 \sigma^2 u - R^2 \sigma$, and $W = e^{-\sigma u} R^{-1}$ (with $\sigma = r_0^{-2} \sqrt{p - r_0^2 \dot{f}^2}$) places the metric in a form which exactly matches the metric[6]

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[5] This is the string metric of the S-dual solution where the RR three form is exchanged for the NS three form.

[6] Note, however, that we are now in the coordinate range $U > 0$ which describes the interior solution, whereas [14] worked with the exterior solution where $U < 0$. This is just the analytic continuation referred to in section II.

in appendix A of [14] when $k = \Delta = a_i = 0$. Since the metric $ds_0$ corresponding to $\Delta = k = a_i = 0$ is already known to be smooth, it is sufficient to analyze the deviations from this metric. Expanding out the solution we find

$$ds^2 - ds_0^2 = \frac{r_0^2 W^2}{2\sigma U}\left(-r_0^2\sigma^2\Delta + k + \Delta\dot{f}^2 - 2a_i\dot{f}^i\right) + higher\ order\ in\ U. \tag{A4}$$

Since $\Delta \sim C_\Delta r^\delta \sim C_\Delta W^\delta U^{\delta/2}$, $k \sim C_k r^\kappa \sim C_k W^\kappa U^{\kappa/2}$, and $a_i \sim C_{a_i} r^{\alpha_i} \sim C_{a_i} W_i^\alpha U^{\alpha_i/2}$ and the coefficients $C_\Delta$, $C_k$, and $C_{a_i}$ are bounded (but not continuous) at $U = 0$, this metric is $C^\epsilon$ for any $\epsilon < \frac{\delta}{2} - 1, \frac{\kappa}{2} - 1, \frac{\alpha_i}{2} - 1$. Similarly, $e^\phi$ is $C^{\gamma/2}$ for all $\gamma < \frac{\delta}{2}$, so the physical (Einstein) metric is again $C^\epsilon$ for any $\epsilon < \frac{\delta}{2} - 1, \frac{\kappa}{2} - 1, \frac{a_i}{2} - 1$.

## APPENDIX B: MOTION OF TEST STRINGS

In this appendix we study the motion of an oscillating test D-string falling into an extremal black hole. We will assume for simplicity that $r_1 = r_5 = r_0$ and that the black hole itself is not carrying any waves. Then the black hole metric can be written in the form (2.8)

$$ds^2 = -F(r)dudv + \frac{p}{r^2}du^2 + F^{-2}(r)dr^2 + r^2 d\Omega_3 + dy_i dy^i \tag{B1}$$

where

$$F(r) \equiv 1 - \left(\frac{r_0}{r}\right)^2 \tag{B2}$$

and $p$ is constant. In addition, $B_{uv} = -F$, and there is a nonzero $B_{\mu\nu}$ on the three sphere which will not play a role in our discussion. The radial coordinate here (which was denoted $\hat{r}$ in (2.8)) is different from that used in most of this paper. The horizon is now at $r = r_0$ and the singularity is at $r = 0$. These coordinates are convenient since they cover both the regions inside and outside the horizon. Thus we will be able to follow the motion of the test string across the horizon.

The motion of D-branes is described by a Dirac-Born-Infield action. For onebranes, this is the same as the usual string action. To begin, we will assume that there is no motion in the angular directions. We will include nonzero angular momentum later. If we use conformal gauge, and introduce null coordinates on the worldsheet $\sigma_\pm = \tau \pm \sigma$, the sigma model action takes the form

$$S \propto \int d\sigma_+ d\sigma_- \left(-F\partial_+ u\partial_- v + \frac{p}{r^2}\partial_+ u\partial_- u + F^{-2}\partial_+ r\partial_- r + \partial_+ y_i \partial_- y^i\right) \tag{B3}$$

Notice that the action is invariant under shifting $v$ by an arbitrary function of $\sigma_+$. This is a direct result of the null Killing vector and the fact that $B_{uv} = g_{uv}$ [22]. In addition to the equations of motion following from this action, we must satisfy the constraints

$$-F\partial_+ u\partial_+ v + \frac{p}{r^2}(\partial_+ u)^2 + F^{-2}(\partial_+ r)^2 + (\partial_+ y_i)^2 = 0 \tag{B4}$$

$$- F\partial_- u \partial_- v + \frac{p}{r^2}(\partial_- u)^2 + F^{-2}(\partial_- r)^2 + (\partial_- y_i)^2 = 0 \tag{B5}$$

The equation of motion for $y^i(\sigma_+, \sigma_-)$ is a simple wave equation, $\partial^2 y^i = 0$. Since this equation decouples from the remaining equations of motion, it follows immediately that the wave on the test string is independent of its other motion. In particular, if the test string falls into the black string, it retains the same wave it had outside. We will assume that the string carries a right moving wave only: $y^i = y^i(\sigma_-)$.

The $v$ equation of motion is $\partial_-(F\partial_+ u) = 0$ which implies that $F\partial_+ u$ is an arbitrary function of $\sigma_+$. In conformal gauge, one has residual gauge freedom to reparameterize $\sigma_\pm$ separately. Using this, one can set $F\partial_+ u$ equal to a constant, which we choose to write as $E/2$. Thus

$$\partial_+ u = \frac{E}{2F} \tag{B6}$$

Using this, the constraint (B4) becomes

$$\partial_+ v = \frac{pE}{2r^2F^2} + \frac{2(\partial_+ r)^2}{EF^2} \tag{B7}$$

The $u$ equation of motion is

$$\partial_+(F\partial_- v) - \partial_+\left(\frac{p}{r^2}\partial_- u\right) - \partial_-\left(\frac{p}{r^2}\partial_+ u\right) = 0 \tag{B8}$$

We will look for a solution where $r$ is a function of $\tau = (\sigma_+ + \sigma_-)/2$ only. Then the $\partial_-$ in the last term above can be replaced by $\partial_+$ (since it acts on a function of $\tau$) and we can immediately integrate to obtain

$$F\partial_- v - \frac{p}{r^2}(\partial_- u + \partial_+ u) = f(\sigma_-) + c \tag{B9}$$

where $f$ is an arbitrary function of $\sigma_-$ which integrates to zero and $c$ is a constant.

Since $r$ is a function of $\tau$ only, we can relate $\partial_- u$ to $\partial_+ u$, and $\partial_- v$ to $\partial_+ v$ by applying $\partial_\sigma = \partial_+ - \partial_-$ to (B6) and (B7) to find $\partial_\sigma \partial_+ u = 0$, $\partial_\sigma \partial_+ v = 0$. So $\partial_\sigma u$ and $\partial_\sigma v$ are both functions of $\sigma_-$ only. One might think that the integral of these functions must vanish. However recall that our spacetime is identified so that $v - u = 2z$ is periodic, and we want our test string to wrap around this compact direction. This implies

$$\partial_- u = \partial_+ u + k + g(\sigma_-) \qquad \partial_- v = \partial_+ v - k + h(\sigma_-) \tag{B10}$$

where both $g$ and $h$ integrate to zero and $k$ is a constant related to the winding number.

Rather than solve the radial equation directly, we use the constraint to obtain an energy-like equation. (This is similar to how one describes the motion of geodesics in spherically symmetric spacetimes.) Substituting (B10) into (B7) and (B9) we obtain two expressions for $\partial_- v$. Setting the $\sigma_-$ dependent terms equal to each other yields

$$f = h = -\frac{p}{r_0^2} g \tag{B11}$$

The remaining terms yield

$$\dot{r}^2 + V_0(r) = 0 \tag{B12}$$

where

$$V_0(r) = -E\left[2kF\left(F + \frac{p}{r^2}\right) + \frac{pE}{r^2} + 2cF\right] \tag{B13}$$

To complete the solution, we now consider the second constraint which we take to be the difference between (B5) and (B4). Since $r$ is a function of $\tau$ only, the $(\partial r)^2$ terms cancel, and using the above results we get

$$g^2 + g\left(k + \frac{E}{2} - \frac{cr_0^2}{p}\right) + \frac{r_0^2}{p}\left(\frac{kE}{2} - ck + (\partial_- y^i)^2\right) = 0 \tag{B14}$$

Notice that all $r$ dependent terms have dropped out, and we are left with a quadratic equation which determines $g(\sigma_-)$ in terms of the wave $(\partial_- y^i)^2$. Since we have assumed that the integral of $g$ vanishes, we determine the arbitary constant $c$ by demanding that this is the case.

The radial motion of the oscillating test string is completely determined by the potential (B13). At infinity, $\dot{r}^2 = 2E(c + k)$, so $E$ is related to the initial kinetic energy of the string. The only way that the oscillations affect the radial motion is through the constant $c$ in the potential. (The constant $k$ is determined by the winding number.) In the absence of waves $y^i(\sigma_-) = 0$, the solution to (B14) is $g(\sigma_-) = 0$ and $c = E/2$. The potential then becomes

$$V_0(r) = -E\left[\left(F + \frac{p}{r^2}\right)(2kF + E)\right] \tag{B15}$$

For positive $E$, this potential is strictly negative outside the horizon, so the test string falls into the black hole. However, it doesn't reach the singularity. The potential vanishes at $r^2 = r_0^2 - p$ and at $r^2 = 2kr_0^2/(2k + E)$, so the test string turns around at the larger of these two values. The first corresponds to the location where $\partial/\partial z$ becomes null. For small $E$ (or large $p$), the second value is larger, and the test string does not penetrate very far inside the horizon.

Since the potential is proportional to $E$, it would appear that if $E = 0$, every configuration of constant $r$ is a solution. However, the derivation assumed $E \neq 0$. In light of the comments in section III concerning the difference between static sources inside and outside the horizon, it is of interest to study this case more closely. For simplicity, we will assume there are no waves $y^i(\sigma_-) = 0$. If $E = 0$, then $\partial_+ u = 0$, so $u = k\sigma_-$. Trying the solution $v = \alpha\sigma_+ + \beta\sigma_-$, constraint (B5) implies $\beta = pk/r^2F$, and (B10) implies

$$\alpha = \beta + k = \frac{k}{F}\left(F + \frac{p}{r^2}\right) \tag{B16}$$

One can easily check that the remaining equations of motion and constraints are satisfied. Thus we do have a solution for a static onebrane at each value of $r$ for which $F \neq 0$ (that is, away from the horizon). However, as one might expect from the discussion in section III, our static solutions describe positively charged onebranes outside the horizon and *negatively* charged onebranes inside the horizon. This follows from the fact that the orientation on the $(u, v)$ plane is $du \wedge dv = k\alpha d\sigma_- \wedge d\sigma_+$. The sign of the onebrane charge is thus determined by the sign of $\alpha$. As long as the onebrane is away from any region of closed timelike curves, $F + \frac{p}{r^2} > 0$, so the sign of the onebrane charge is determined by the sign of $F$, which changes at the horizon.

Nonradial motion can be included just as one does for geodesics. By spherical symmetry, the test string moves in a plane. Let $\varphi$ denote the angle on the plane, and we will assume that it is only a function of $\tau$. Then the field equation is $(r^2\dot{\varphi})^{\cdot} = 0$, which implies $\dot{\varphi} = L/r^2$ for a constant $L$. The constraint (B4) now picks up an extra term $r^2(\partial_+\varphi)^2$ so that the radial equation becomes $\dot{r}^2 + V = 0$ where

$$V = V_0 + F^2 \frac{L^2}{r^2} \tag{B17}$$

and $V_0$ is the potential (B13) with no angular momentum. The angular momentum barrier vanishes near at the horizon, but is positive outside. So for a given energy, there is a range of angular momentum which will still result in capture by the black hole.

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